EXPONENTS

When two or more numbers are multiplied to give a certain product, the numbers multiplied are each called a factor of the product.

For example, $5 \text{ x } 6 = 30$ (5 and 6 are called factors of the product 30)

If the same factor is repeated more than once to form a product, the factor can be written with an exponent.

For example, the product 48 can be written as:

4 x 4 x 3 = 4^2 x 3 (4 is called the base and the exponent 2 indicates that 4 is a factor twice)

$$(a)(a) = a^2$$
$$(a)(a)(a) = a^3$$
$$(a)(a)(a)....(a) = \text{a}^n \text{ (where there are "n" factors of "}a\text{")}$$

LAWS OF EXPONENTS (where "a" is any number and "n" and "m" are a positive integer exponents)

1. Multiplication Law: $a^n . a^m = a^{n+m}$
2. Division Law: $\frac{a^n}{a^m} = a^{n-m}$ (where $a \neq 0$ and n > m)
3. Power Law: $(a^n)^m = a^{nm}$

LAWS OF EXPONENTS (for exponent $n = 0$) $a^n = a^0 = 1$

Proof:

$$a^n . a^0 = a^{n+0} = a^n \quad \text{(Multiplication Law for Exponents)}$$
$$a^n . 1 = a^n \quad \text{(Multiplicative Identity)}$$
$$a^n . a^0 = a^n . 1 \quad \text{(Quantities equal to the same quantity are equal to each other)}$$
$$\therefore\ a^0 = 1$$

LAWS OF EXPONENTS (for negative exponent, n a positive number) $a^{-n} = \frac{1}{a^n}$

Proof:

$$a^n . a^{-n} = a^0 = 1 \quad \text{(Multiplication Law for Exponents)}$$
$$a^n . \frac{1}{a^n} = 1 \quad \text{(Multiplying a number by it's inverse)}$$
$$a^n . a^{-n} = a^n . \frac{1}{a^n} \quad \text{(Quantities equal to same quantity are equal to each other)}$$
$$\therefore\ a^{-n} = \frac{1}{a^n}$$

FRACTIONAL EXPONENTS

The n^{th} root of a number is defined as one of the n equal factors of the number. Thus, if x is a positive number, Then

$\sqrt[n]{x^n} = x$ **(Definition of n^{th} root of a number)**

However,

$\left(x^n\right)^{\frac{1}{n}} = x^{\frac{n}{n}} = x$ **(Power Law for exponents)**

Therefore,

$\sqrt[n]{x^n} = (x^n)^{\frac{1}{n}}$ **(Quantities equal to the same quantity are equal to each other)**

This last equation implies that another way of writing the n^{th} root of x $(\sqrt[n]{x})$ is by using a fractional exponent $(x^{\frac{1}{n}})$.