# Exponential and Logarithmic Functions

## The Natural Logarithmic Function

1. The Natural logarithmic function is defined by:

$$\ln x = \int_1^x \frac{1}{t} dt \qquad \text{Where, } x > 0$$

2. The domain of the natural logarithmic function is the set of all positive real numbers.

3. The derivative of the natural logarithmic function is obtained by the application of **the Second Fundamental Theorem of Calculus** as follows:

$$\frac{d}{dx}[\ln x] = \frac{d}{dx}\left[\int_1^x \frac{1}{t} dt\right] = \frac{1}{x}$$

4. The letter $e$ denotes the positive real number such that:

$$\ln e = \int_1^e \frac{1}{t} dt = 1$$

5. The natural logarithmic function $f(x) = \ln x$ is increasing (monotonic) on its entire domain and therefore, has an inverse.

## The Natural Exponential Function

1. The inverse of the natural logarithmic function $f(x) = \ln x$ is called the **natural exponential function** and mathematically expressed as:

$$f^{-1}(x) = e^x$$

2. The derivative of the natural exponential function is:

$$\frac{d}{dx}\left[e^x\right] = e^x$$

3. When an integral involves a base other than $e$, there are two options:
   a) Convert to a base $e$ using the formula: $b^x = e^{(\ln b)x}$ and then integrate
   b) Integrate directly using the formula $\int b^x dx = \left(\frac{1}{\ln b}\right) b^x + C$