

## Integration by Parts

To find the anti-derivative of integrals involving products of the type  $x^n e^{ax}$ ,  $x^n \sin(ax)$ ,  $x^n \cos(ax)$  with  $n$  a positive integer and  $a$  a constant, use the following **integration by parts** formula:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

When using this integration formula for these types of products, Integrate by parts  $n$  times, differentiating the power of  $x$  and integrating the  $e^{ax}$ ,  $\sin(ax)$ , or  $\cos(ax)$  each time.

The anti-derivative for some integrals involving  $\ln(x)$ ,  $\arcsin(x)$ ,  $\arctan(x)$  can be found using the **u-substitution method** ( $u = \ln(x)$ ,  $u = \arcsin(x)$ , or  $u = \arctan(x)$ ). The Anti-derivative for other such integrals can be found by **integration by parts** in which the function  $\ln(x)$ ,  $\arcsin(x)$ , or  $\arctan(x)$  is differentiated.

Integrate by *parts* **twice** when integrating integrals of the type:

$$\begin{array}{ll} \sin(ax) \sin(bx) \text{ or } \cos(ax) \cos(bx) & \text{with } a^2 \neq b^2 \\ e^{ax} \sin(bx), e^{ax} \cos(bx), \text{ or } \sin(ax) \cos(bx) & \text{with arbitrary constants } a \text{ and } b \end{array}$$