Some Laws of Logic

1. Law of Detachment

   If the conditional $p \to q$ is true and the hypothesis of the conditional (p) is true, then the conclusion (q) of the conditional is true.

   Written symbolically:

   $$\begin{array}{l} p \to q \\ \underline{p} \\ \therefore q \end{array}$$

2. Law of Contrapositive

   If the conditional $p \to q$ is true then, the contrapositive of the conditional $\sim q \to \sim p$ is true.

3. Law of Modulus Tollens

   If the conditional $p \to q$ is true and negation of the conclusion of the conditional is true, then the negation of the hypothesis is true.

   Written symbolically:

   $$\begin{array}{l} p \to q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$

4. Chain rule

   If $p \to q$ and $q \to r$ are two true conditionals such that the conclusion of the first is the hypothesis of the second, then the conditional $p \to r$ formed by connecting the hypothesis of the first conditional and the conclusion of the second conditional is also true.

   Written symbolically

   $$\begin{array}{l} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

5. Law of Disjunctive Inference

If the disjunctive $b \vee n$ is true and the negation of one of the disjuncts is true, then the other disjunct is true.

Written symbolically:

$$\begin{array}{l} b \vee n \\ \sim n \\ \hline \therefore b \end{array}$$

6. DeMorgan's Law

- Negation of a conjunction

  The negation of a conjunction of two statements is logically equivalent to the disjunction of the negation of the two statements.

  Written symbolically:

  $$\sim(p \wedge r) \leftrightarrow \sim p \vee \sim r$$

- Negation of a disjunction

  The negation of a disjunction of two statements is logically equivalent to the conjunction of the negation of the two statements.

  Written symbolically:

  $$\sim(p \vee r) \leftrightarrow \sim p \wedge \sim r$$

7. Law of Simplification

   If a single conjunctive premise is true, it follows that each of the individual conjuncts must be true:

   Written symbolically:

   If $(p \wedge q)$ is true, then

   $p \wedge q \rightarrow p$ or $p \wedge q \rightarrow q$

8. Law of Conjunction

   When two given premises are true, the conjunction of these premises is true.

   Written symbolically:

   If p is true and q is true, then

   $(p \wedge q)$ is true.