Fibonacci Numbers

The first two numbers in the fibonacci sequence is the number 1. Subsequent numbers in the sequence are obtained by adding the previous two numbers. Thus, the sequence is as follows:

1,1,2,3,5,8,13,21,34, …..

Expressing this mathematically, the nth fibonacci number (starting with n=3) is given by:

$$F_n = F_{n-2} + F_{n-1}$$

Notes on the Fibonacci sequence

1. The sum of the **1st n** fibonacci numbers is given by:
$$\sum_1^n F_i = F_{n+2} - 1$$
2. The sum of the **1st (n+1) even** fibonacci numbers plus one is the fibonacci number F_{2n+1}. Stated mathematically:
$$F_{2n+1} = \sum_{i=1}^{n+1} F_{2i} + 1$$
3. The sum of the **1st n odd** fibonacci numbers is the fibonacci number F_{2n}. Stated mathematically:
$$F_{2n} = \sum_{i=1}^{n} F_{2i-1}$$
4. Every third fibonacci number is even.
5. Every 4th fibonacci number is a multiple of 3.
6. Every 5th fibonacci number is a multiply of 5.
7. Every 6th fibonacci number is a multiple of 8.
8. Theorem – The fibonacci number F_m divides fibonacci number F_n if and only if m divides n.
9. The greatest common denominator (GCD) between two fibonacci numbers (F_m, F_n) is the fibonacci number $F_{GCD(m,n)}$

10. If m is a composite number (except for $m = 4$), then the fibonacci number F_m is guaranteed to be composite.

11. Every prime number (P) divides a Fibonacci number.

- If the prime number P ends in 1 or 9, then P divides the fibonacci number F_{P-1}.

- If the prime number P ends in 3 or 7, then P divides the fibonacci number F_{p+1}

12. The sum of the squares of **two consecutive** fibonacci numbers is always a fibonacci number. Stated mathematically:

$$(F_n)^2 + (F_{n+1})^2 = F_{2n+1}$$

13. The sum of the squares of n fibonacci numbers is given by:

$$\sum_{i=1}^{n} (F_i)^2 = F_n \ x \ F_{n+1}$$

14. Two relationships involving the neighbors around a fibonacci number are:

- $(F_n)^2 + (-1)^n = F_{n-1} \ x \ F_{n+1}$
- $(F_n)^2 + (-1)^n = F_{n-2} \ x \ F_{n+2}$

15. The ratio of consecutive fibonacci numbers seems to converge on what is known as the golden ratio [$\frac{(1+\sqrt{5})}{2}$].

16. The relation ship between the golden ratio and fibonacci numbers can be expressed mathematically using Binet's Formula:

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$