Sinusoidal Function

Problem: Find values of $a, b,$ and h (to the nearest hundredth) so that $f(x) = a\sin[b(x-h)]$

Given: $f(x) = 5\sin(4x) - 3\cos(4x)$

Step 1 - Rewrite $f(x) = a\sin[b(x-h)]$ as follows:

$$f(x) = a\sin(bx - bh)$$

This implies we need to use the following trigonometric identity:

$$sin(A-B) = sin(A)\cos(B) - cos(A)\sin(B)$$

where, $A = bx, B = bh$

Step 2 - Apply the trigonometric formula to once again rewrite the function

$$f(x) = a\sin(bx - bh) = a\sin(bx)\cos(bh) - a\cos(bx)\sin(bh)$$

$$= [a\cos(bh)]\sin(bx) - [a\sin(bh)]\cos(bx)$$

Step 3 - Equate the given function with the expression obtained in step 2.

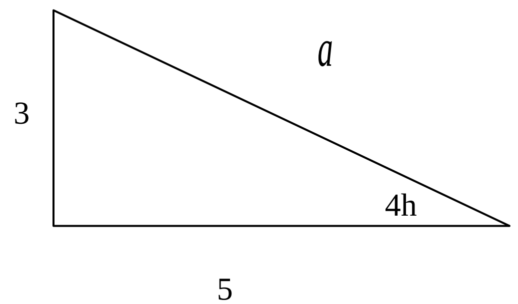
$$5\sin(4x) - 3\cos(4x) = [a\cos(bh)]\sin(bx) - [a\sin(bh)]\cos(bx)$$

Comparing coefficients gives us the following:

$a\cos(bh) = 5,\ a\sin(bh) = 3,\ b = 4$

$\therefore$ since $b = 4,$ we have: $\cos(4h) = 5/a$ and $\sin(4h) = 3/a$

Step 4 – Using Pythagorean theorem, $\cos(4h) = 5/a$ and $\sin(4h) = 3/a$ with the following right triangle we can now determine the value of a.



$$a^2 = 5^2 + 3^2$$
$$a^2 = 25 + 9$$
$$a^2 = 34$$
$$a = \pm\sqrt{34}$$

Step 5 – Determine the value of h.

Since the $\sin(4h) = 3/a$ and $a = \sqrt{34}$,
the $\sin(4h) = 3/\sqrt{34}$

Using the inverse trigonometric function on your calculator we see that
$4h \approx 30.9638$ therefore, $h \approx 7.741$

$\therefore$ **ANSWER**

$$a = \sqrt{34}$$
$$b = 4$$
$$h = 7.74$$

Step 6 – Check your answer

You can check your answer by arbitrarily choosing a value for x (an angle in degrees) and evaluate the given function $f(x) = 5\sin(4x) - 3\cos(4x)$.

Then using the same value of x evaluate the following function:
$f(x) = a\sin(bx - bh) = \sqrt{34}\sin[4x - 4(7.74)]$

The two functions should have approximately the same value for $f(x)$. In other words, $5\sin(4x) - 3\cos(4x) \approx \sqrt{34}\sin[4x - 4(7.74)]$ for any arbitrarily chosen value of x.

Notice that the function on the right of the approximate symbol uses the sine function only.