

Trigonometric Identities

Reciprocals of trigonometric functions:

- $\sin \theta = \frac{1}{\csc \theta}$ $\csc \theta = \frac{1}{\sin \theta}$
- $\cos \theta = \frac{1}{\sec \theta}$ $\sec \theta = \frac{1}{\cos \theta}$
- $\tan \theta = \frac{1}{\cot \theta}$ $\cot \theta = \frac{1}{\tan \theta}$

Trigonometric functions of negative angles:

- $\sin(-\theta) = -\sin(\theta)$
- $\cos(-\theta) = \cos(\theta)$
- $\tan(-\theta) = -\tan(\theta)$
- $\csc(-\theta) = -\csc(\theta)$
- $\sec(-\theta) = \sec(\theta)$
- $\cot(-\theta) = -\cot(\theta)$

Some common trigonometric identities:

- $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\cot \theta = \frac{\cos \theta}{\sin \theta}$
- $\sin^2 \theta + \cos^2 \theta = 1$
- $1 + \tan^2 \theta = \sec^2 \theta$
- $1 + \cot^2 \theta = \csc^2 \theta$

Identities involving the sum and difference of two angles:

- $\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
- $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$
- $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
- $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
- $\tan(A + B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}$
- $\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$

Double angle identities:

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
- $\cos(2\theta) = 1 - 2\sin^2(\theta)$
- $\cos(2\theta) = 2\cos^2(\theta) - 1$
- $\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$

Half angle identities:

- $\sin\left(\frac{1}{2}\theta\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$
- $\cos\left(\frac{1}{2}\theta\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$
- $\tan\left(\frac{1}{2}\theta\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}$

Special angle identities ($90 - \theta$):

- $\sin(90 - \theta) = \cos(\theta)$
- $\cos(90 - \theta) = \sin(\theta)$
- $\tan(90 - \theta) = \cot(\theta)$
- $\sec(90 - \theta) = \csc(\theta)$
- $\csc(90 - \theta) = \sec(\theta)$
- $\cot(90 - \theta) = \tan(\theta)$

Special angle identities $(180-\theta)$, $(180+\theta)$, $(360-\theta)$:

- $\sin(180-\theta) = \sin \theta$ $\sin(180+\theta) = -\sin \theta$ $\sin(360-\theta) = -\sin(\theta)$
- $\cos(180-\theta) = -\cos(\theta)$ $\cos(180+\theta) = -\cos(\theta)$ $\cos(360-\theta) = \cos(\theta)$
- $\tan(180-\theta) = -\tan(\theta)$ $\tan(180+\theta) = \tan(\theta)$ $\tan(360-\theta) = -\tan(\theta)$

Products of trigonometric functions in terms of sums:

- $\sin(A)\cos(B) = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$
- $\cos(A)\sin(B) = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$
- $\cos(A)\cos(B) = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$
- $\sin(A)\sin(B) = \frac{1}{2}[\cos(A+B) - \cos(A-B)]$

Sums of trigonometric functions in terms of products:

- $\sin(A) + \sin(B) = 2\sin\left[\frac{1}{2}(A+B)\right]\cos\left[\frac{1}{2}(A-B)\right]$
- $\sin(A) - \sin(B) = 2\cos\left[\frac{1}{2}(A+B)\right]\sin\left[\frac{1}{2}(A-B)\right]$
- $\cos(A) + \cos(B) = 2\cos\left[\frac{1}{2}(A+B)\right]\cos\left[\frac{1}{2}(A-B)\right]$
- $\cos(A) - \cos(B) = -2\sin\left[\frac{1}{2}(A+B)\right]\sin\left[\frac{1}{2}(A-B)\right]$

Hyperbolic Identities

- $\cosh^2 x - \sinh^2 x = 1$
- $\tanh^2 x + \operatorname{sech}^2 x = 1$
- $\coth^2 x - \operatorname{csch}^2 x = 1$
- $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$
- $\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$
- $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$
- $\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$
- $\sinh^2 x = \frac{-1 + \cosh 2x}{2}$ $\cosh^2 x = \frac{1 + \cosh 2x}{2}$
- $\sinh 2x = 2 \sinh x \cosh x$ $\cosh 2x = \cosh^2 x + \sinh^2 x$